

CAN BLACK HOLE RELAX UNITARILY?

SERGEY N. SOLODUKHIN

*School of Engineering and Science,
International University Bremen,
P.O. Box 750561, Bremen 28759, Germany
E-mail: s.solodukhin@iu-bremen.de*

We review the way the BTZ black hole relaxes back to thermal equilibrium after a small perturbation and how it is seen in the boundary (finite volume) CFT. The unitarity requires the relaxation to be quasi-periodic. It is preserved in the CFT but is not obvious in the case of the semiclassical black hole the relaxation of which is driven by complex quasi-normal modes. We discuss two ways of modifying the semiclassical black hole geometry to maintain unitarity: the (fractal) brick wall and the worm-hole modification. In the latter case the entropy comes out correctly as well.

1. Introduction

Any thermodynamical system initially in equilibrium at finite temperature and then perturbed tends to return to the equilibrium if the perturbation is not too big and does not last too long. The important parameter which characterizes this process is the relaxation time τ ¹. In fact, the process of relaxation back to the equilibrium is a particular and most easily tractable example of more general phenomenon of the thermalization, when system initially far from being thermal gets thermalized to a state characterized by certain temperature. The way how the thermalization goes for different systems is an important and still poorly understood problem. Gravitational physics gives yet another example of system which behaves thermally. This system is black hole. The black hole formation can be viewed as another example of the thermalization: non-thermal collapsing body and the flat space-time geometry in the beginning transform to (thermal) black hole state in the end of the gravitational collapse. On the other hand, the already formed and stayed in equilibrium black hole can be perturbed by exciting a pulse of matter field in the exterior of black hole. The subsequent relaxation is very well studied in the literature and is known to be characterized by the so-called quasi-normal modes. These modes are eigen values of the radial

Schrödinger type equation subject to certain boundary conditions. These are dissipative boundary conditions saying that the perturbation should leave the region through all possible boundaries. In general there are two such boundaries: black hole horizon and spatial infinity. Formulated this way the boundary value problem is not self-adjoint so that the quasi-normal modes are typically complex $\omega = \omega_R - i\omega_I$ with negative imaginary part. In most cases there is discrete set of such modes parameterized by integer number n . The imaginary value of the lowest ($n = 1$) quasi-normal mode sets the relaxation time $\tau = 1/\omega_I$.

In fact what we have said in the beginning about the relaxation of any system back to thermal equilibrium should be made more precise: the system should be in infinite volume. In finite volume and if the evolution of the system is unitary, any perturbation once created never leaves the system so that the complete returning to the initial unperturbed state is not possible. Thus information about the perturbation never disappears completely and always can be restored. The characteristic time during which the perturbation (as well as the whole state of the system) is guaranteed to come back is set by the Poincaré recurrence time. All this means that the characteristic frequencies which run the perturbation of unitary system in finite volume should be real and discrete. Depending on these frequencies the evolution of the perturbation is quasi-periodic or even chaotic. But it can never be dissipative. Thus, strictly speaking for the thermalization we need infinite volume. Of course, nothing is infinite in the real world. The system still may be considered as thermal during the interval of time which is considerably less than the Poincaré recurrence time.

What this implies for the black holes? More specifically, for asymptotically AdS black holes the state of thermal equilibrium of which is well defined and can last infinitely long? Such a black hole can be viewed as system put in the box with the size set by the AdS radius. So one would have to expect this black hole to behave as any other system in the finite volume and in particular to show the Poincaré recurrences (for the discussion of this in de Sitter space see ³). This however does not happen in the semiclassical black hole: the complex quasi-normal modes are always there. The presence of these modes is related to the very existence of the horizon. Once there is black hole horizon there will always be complex frequencies which govern the time evolution of the perturbation. This problem is a manifestation of the long-time debated issue of whether the black hole evolution is actually unitary (see ² and ³).

A refreshed look at the whole issue is offered by the AdS/CFT corre-

spondence (see review in ⁴). According to this correspondence the gravitational physics in the bulk of asymptotically AdS space has a dual description in terms of a Conformal Field Theory (CFT) living on the boundary. Thus, the black hole in the bulk corresponds to a thermal CFT. The relaxation of the black hole then has a dual description as relaxation of the CFT after a perturbation driven by certain conformal operator has been applied to the system. The quasi-normal modes thus set the time scale for the relaxation in the boundary CFT ⁵. The effect of the finite size however is rather delicate issue. It has been studied in paper ⁶ and is reviewed in section 3 of this note. The similar conclusions have been made in paper ⁷. The recent reviews on the issue of black hole relaxation and unitarity are ⁸ and ⁹.

2. Relaxation in black hole: quasi-normal modes

We consider (2+1)-dimensional BTZ black hole with metric given by

$$ds^2 = -\sinh^2 y \, dt^2 + dy^2 + \cosh^2 y \, d\phi^2, \quad (1)$$

where for simplicity we consider non-rotating black hole and set the size of the horizon $r_+ = 1$ and AdS radius $l = 1$. The coordinate ϕ is periodic with period L so that the boundary has topology of cylinder and L sets the finite size for the boundary system. A bulk perturbation $\Phi_{(m,s)}$ of mass m and spin s should satisfy the quasi-normal boundary condition, i.e. it should be in-going at the horizon and have vanishing flux at the infinity. The latter condition comes from the fact that in the asymptotically AdS space-times the effective radial potential is growing at infinity so that there can be no propagating modes as well as no leakage of the energy through the boundary. The relevant radial equation takes the form of the hypergeometric equation which is exactly solvable. The quasi-normal modes in general fall into two sets ^{10,11}

$$\begin{aligned} \omega &= \frac{2\pi}{L}m - 4\pi iT_L(n + \bar{h}) \\ \omega &= -\frac{2\pi}{L}m - 4\pi iT_R(n + h), \quad m \in \mathbf{Z}, n \in \mathbf{N} \end{aligned} \quad (2)$$

where the left- and right-temperatures $T_L = T_R = 1/2\pi$ and (h, \bar{h}) have the meaning of the conformal weights of the dual operator $\mathcal{O}_{(h, \bar{h})}$ corresponding to the bulk perturbation $\Phi_{(m,s)}$, with $h + \bar{h} = \Delta(m)$, $h - \bar{h} = s$ and $\Delta(m)$ is determined in terms of the mass m .

For comparison, in the case of global anti-de Sitter space the horizon and respectively the quasi-normal modes are absent. But, instead, one can

define the normalizable modes which form a discrete set of real frequencies¹⁸

$$\omega = 2\pi m/L + 4\pi(n+h)/L \quad n \in \mathbf{N} \quad (3)$$

where the size of the boundary is also set to be L as in the black hole case.

3. Relaxation in CFT₂

The thermal state of the black hole in the bulk corresponds to the thermal state on the CFT side. In fact, the boundary CFT factorizes on left- and right-moving sectors with temperature T_L and T_R respectively. The bulk perturbation corresponds to perturbing the thermal field theory state with operator $\mathcal{O}_{(h,\bar{h})}$. The further evolution of the system is described by the so-called Linear Response Theory (see ¹). According to this theory one has to look at the time evolution of the perturbation itself. More precisely, the relevant information is contained in the retarded correlation function of the perturbation at the moments t and $t = 0$ (when the perturbation has been first applied). Since the perturbation is considered to be small the main evolution is still governed by the unperturbed Hamiltonian over the thermal state so that the correlation function is the thermal function at temperature T . Thus, the analysis boils down to the study of the thermal 2-point function of certain conformal operators. Such a function should be double periodic: with period $1/T$ in the direction of the Euclidean time and with period L in the direction of the compact coordinate ϕ . This can be first calculated as a 2-point function on the Euclidean torus and then analytically continued to the real time.

3.1. Universality

In general the correlation function on torus can be rather complicated since its form is not fixed by the conformal symmetry. The conformal symmetry however may help to deduce the universal form of the 2-point function in two special cases: when size L of the system is infinite (temperature T is kept finite) and when inverse temperature is infinite (the size L is finite). The universal form of the (real time) 2-point function in the first case is

$$\langle \mathcal{O}(t, \phi) \mathcal{O}(0, 0) \rangle = \frac{(\pi T)^{2(h+\bar{h})}}{(\sinh \pi T(\phi - t))^{2h} (\sinh \pi T(\phi + t))^{2\bar{h}}} \quad (4)$$

which for large t decays exponentially as $e^{-2\pi T(h+\bar{h})t}$. The information about the perturbation is thus lost after characteristic time set by the inverse temperature. It is clear that this happens because in infinite volume

the information may dissipate to infinity. In the second case correlator

$$\langle \mathcal{O}(t, \phi) \mathcal{O}(0, 0) \rangle = \frac{(\pi/iL)^{2(h+\bar{h})}}{(\sin \frac{\pi}{L}(t+\phi))^{2h} (\sin \frac{\pi}{L}(t-\phi))^{2\bar{h}}} \quad (5)$$

has the oscillatory behavior. Notice that the oscillatory behavior in the second case should have been expected since the system lives on the circle. The perturbation once created at the moment $t = 0$ at the point $\phi = 0$ travels around the circle with the speed of light and comes back to the same point at $t = L$. Thus, the information about the perturbation is never lost. The correlation function (5) as a function of time represents a series of singular picks concentrated at $t = \pm\phi + nL$, $n \in \mathbf{N}$. In fact, this behavior should be typical for any system with unitary evolution in finite volume.

It is interesting to see what happens in the intermediate regime when both L and $1/T$ are kept finite. In this case the behavior of the correlation functions is not universal, may depend on the (self)interaction in the system and is known only in some cases. We consider two instructive examples: the free fermion field and the strongly coupled CFT which is dual to the gravity on AdS_3 .

3.2. Intermediate regime: Free fermions

The two point function of free fermions on the torus is known explicitly (e.g. ¹³). The real time correlation function is

$$\langle \psi(w) \psi(0) \rangle_\nu = \frac{\theta_\nu(wT|iLT) \partial_z \theta_1(0|LT)}{\theta_\nu(0|iLT) \theta_1(wT|iLT)}, \quad (6)$$

where $w = i(t + \phi)$ and ν characterizes the boundary conditions for $\psi(w)$. For finite temperature boundary conditions we have $\nu = 3, 4$. Using the properties of θ -functions, it is then easy to see that (6) is invariant under shifts $w \rightarrow w + 1/T$ and $w \rightarrow w + iL$. It is then obvious that the resulting real time correlator (6) is a *periodic* function of t with period L . Zeros of the theta function $\theta_1(wT|iLT)$ are known ¹³ to lie at $w = m/T + inL$, where m, n are arbitrary relative integers. Therefore, for real time t , the correlation function (6) is a sequence of singular peaks located at $(t + \phi) = nL$. Using the standard representation ¹³ of the θ -functions, we also find that in the limit $LT \rightarrow \infty$ the correlation function (6) approaches the left-moving part of (3) with $h = 1/2$ that exponentially decays with time,

$$\langle \psi(w) \psi(0) \rangle_{3(4)} = \frac{\pi T}{4 \sinh \pi T(t + \phi)} [1 \pm 2e^{-\pi LT} \cosh 2\pi T(t + \phi) + \dots] \quad (7)$$

In the opposite limit, when $LT \rightarrow 0$, it approaches the oscillating function (5). A natural question is how the asymptotic behavior (7) when size of the system is taken to infinity can be consistent with the periodicity, $t \rightarrow t + L$, of the correlation function (6) at any finite L ? In order to answer this question we have to observe that there are two different time scales in the game. The first time scale is set by the inverse temperature $\tau_1 = 1/T$ and is kept finite while the second time scale is associated with the size of the system $\tau_2 = 1/L$. When L is taken to infinity we have that $\tau_2 \gg \tau_1$. Now, when the time t is of the order of τ_1 but much less than τ_2 the asymptotic expansion (7) takes place. The corrections to the leading term are multiplied by the factor $e^{-\pi LT}$ and are small. The 2-point function thus is exponentially decaying in this regime. It seems that the system has almost lost information about the initial perturbation (at $t = 0$). But it is not true: as time goes on and approaches the second time scale $t \sim \tau_2$ the corrections to the leading term in (7) become important and the system starts to collect its memory about the initial perturbation. The information is completely recovered as $t = \tau_2$ and the time-periodicity is restored. This example is instructive. In particular, it illustrates our point that there can be thermalization in the finite volume for relatively small intervals of time, i.e. when $t \ll \tau_2$.

3.3. Strongly coupled CFT_2 dual to AdS_3

As an example of a strongly coupled theory we consider the supersymmetric conformal field theory dual to string theory on AdS_3 . This theory describes the low energy excitations of a large number of D1- and D5-branes ⁴. It can be interpreted as a gas of strings that wind around a circle of length L with target space T^4 . The individual strings can be simply- or multiply wound such that the total winding number is $\mathbf{k} = \frac{c}{6}$, where $c \gg 1$ is the central charge. The parameter \mathbf{k} plays the role of N in the usual terminology of large N CFT.

According to the prescription (see ⁴), each AdS space which asymptotically approaches the given two-dimensional manifold should contribute to the calculation, and one thus has to sum over all such spaces. In the case of interest, the two-manifold is a torus (τ, ϕ) , where $1/T$ and L are the respective periods. There exist two obvious AdS spaces which approach the torus asymptotically. The first is the BTZ black hole in AdS_3 and the second is the so-called thermal AdS space, corresponding to anti-de Sitter space filled with thermal radiation. Both spaces can be represented (see ¹⁴)

as a quotient of three dimensional hyperbolic space H^3 , with line element

$$ds^2 = \frac{l^2}{y^2}(dzd\bar{z} + dy^2) \quad y > 0 \quad . \quad (8)$$

In both cases, the boundary of the three-dimensional space is a rectangular torus with periods L and $1/T$. We see that the two configurations (thermal AdS and the BTZ black hole) are T-dual to each other, and are obtained by the interchange of the coordinates $\tau \leftrightarrow \phi$ and $L \leftrightarrow 1/T$ on the torus. In fact there is a whole $SL(2, \mathbf{Z})$ family of spaces which are quotients of the hyperbolic space.

In order to find correlation function of the dual conformal operators, one has to solve the respective bulk field equations subject to Dirichlet boundary condition, substitute the solution into the action and differentiate the action twice with respect to the boundary value of the field. The boundary field thus plays the role of the source for the dual operator $\mathcal{O}_{(h, \bar{h})}$. This way one can obtain the boundary CFT correlation function for each member of the family of asymptotically AdS spaces. The total correlation function is then given by the sum over all $SL(2, \mathbf{Z})$ family with appropriate weight. For our purposes however it is sufficient to consider the contribution of only two contributions¹⁶

$$\langle \mathcal{O}(t, \phi) \mathcal{O}(0, 0) \rangle = e^{-S_{\text{BTZ}}} \langle \mathcal{O} \mathcal{O}' \rangle_{\text{BTZ}} + e^{-S_{\text{AdS}}} \langle \mathcal{O} \mathcal{O}' \rangle_{\text{AdS}} \quad , \quad (9)$$

where $S_{\text{BTZ}} = -k\pi LT/2$ and $S_{\text{AdS}} = -k\pi/2LT$ are Euclidean action of the BTZ black hole and thermal AdS₃, respectively¹⁵. On the Euclidean torus $\langle \rangle_{\text{BTZ}}$ and $\langle \rangle_{\text{AdS}}$ are T-dual to each other. Their exact form can be computed explicitly¹⁷. For our purposes it is sufficient to note that the (real-time) 2-point function coming from the BTZ part is exponentially decaying, $\langle \rangle_{\text{BTZ}} \sim e^{-2\pi h T t}$ even though it is a correlation function in a system of finite size L . On the other hand, the part coming from the thermal AdS is oscillating with period L as it should be for a system at finite size. Thus, the total 2-point function (9) has two contributions: one is exponentially decaying and another is oscillating. So that (9) is *not* a quasi-periodic function of time t . This conclusion does not seem to change if we include sum over $SL(2, \mathbf{Z})$ in (9). There will always be contribution of the BTZ black hole that is exponentially decaying. This can be formulated also in terms of the poles in the momentum representation of 2-point function (see¹¹ and¹⁹). The poles of $\langle \rangle_{\text{BTZ}}$ are exactly the complex quasi-normal modes (2) while that of $\langle \rangle_{\text{AdS}}$ are the real normalizable modes (3).

Depending on the value of LT , one of the two terms in (9) dominates¹⁵. For high temperature (LT is large) the BTZ is dominating, while at low

temperature (LT is small) the thermal AdS is dominant. The transition between the two regimes occurs at $1/T = L$. In terms of the gravitational physics, this corresponds to the Hawking-Page phase transition¹². This is a sharp transition for large \mathbf{k} , which is the case when the supergravity description is valid. The Hawking-Page transition is thus a transition between oscillatory relaxation at low temperature and exponential decay at high temperature.

3.4. *The puzzle and resolution*

Thus, the AdS/CFT correspondence predicts that the CFT dual to gravity on AdS_3 is rather peculiar. Even though it is in finite volume the relaxation in this theory is combination of oscillating and exponentially decaying functions. This immediately raises a puzzle: how this behavior is consistent with the general requirement for a unitary theory in finite volume to have only quasi-periodic relaxation? Resolution of this puzzle was suggested in⁶. It was suggested that additionally to the size L there exists another scale in the game. This scale appears due to the fact that in the dual CFT at high temperature the typical configuration consists of multiply wound strings which effectively propagate in a much bigger volume, $L_{\text{eff}} \sim \mathbf{k}L$. The gravity/CFT duality however is valid in the limit of infinite \mathbf{k} in which this second scale becomes infinite. So that the exponential relaxation corresponds to infinite effective size L_{eff} that is in complete agreement with the general arguments. At finite \mathbf{k} the scale L_{eff} would be finite and the correlation function is expected to be quasi-periodic with two periods $1/L$ and $1/L_{\text{eff}}$. The transition of this quasi-periodic function to combination of exponentially decaying and oscillating functions when L_{eff} is infinite then should be similar to what we have observed in the case of free fermions when L was taken to infinity.

4. **Black hole unitarity: finite \mathbf{k}**

That relaxation of black hole is characterized by a set of complex frequencies (quasi-normal modes) is mathematically precise formulation of the lack of unitarity in the semiclassical description of black holes. The unitarity problem was suggested to be resolved within the AdS/CFT correspondence¹⁶. Indeed, the theory on the boundary is unitary and there should be a way of reformulating the processes happening in the bulk of black hole space-time on the intrinsically unitary language of the boundary CFT. The analysis of the relaxation is helpful in understanding how this reformulation

should work. Before making comments on that let us note that the loss of information in semiclassical black hole is indeed visible on the CFT side. It is encoded in that exponentially decaying contribution to the 2-point correlation function. For the CFT itself this however is not a problem. As we discussed above the finite size unitarity is restored at finite value of k . This however goes beyond the limits where the gravity/CFT duality is formulated. Assuming that the duality can be extended to finite k an important question arises: *What would be the gravity counter-part of the duality at finite k ?* Obviously, it can not be a semiclassical black hole. The black hole horizon should be somehow removed so that the complex quasi-normal modes (at infinite k) would be replaced by real (normal) modes when k is finite. Below we consider two possibilities of how it may happen.

4.1. *Fractal brick wall*

It was suggested in ⁷ that the quantum modification of the black hole geometry needed for the restoring the Poincaré recurrences can be modeled by the brick wall. Here we elaborate on this interesting idea. The brick wall is introduced by placing a boundary at small distance ϵ from the horizon and cutting off a part of the space-time lying inside the boundary. The effect of the boundary on the quantum fields is implemented by imposing there the Dirichlet boundary condition. Originally, the brick wall was introduced by 't Hooft ²⁰ for regularizing the entropy of the thermal atmosphere outside black hole horizon. With this regularization the quantum entropy $S_q \sim \frac{A}{\epsilon^{d-2}}$ correctly reproduces the proportionality of the black hole entropy to the horizon area $A \sim r_+^{d-2}$. Assuming that ϵ is taken to be of the order of the Planck length, so that Newton's constant is $G \sim \epsilon^{d-2}$, one can argue that the black hole entropy is correctly reproduced in this approach. Later on it was however realized that the brick wall divergence is actually a UV divergence. One can introduce a set of the Pauli-Villars fields with masses set by parameter μ which plays the role of the UV regulator. Taking into account the contribution of the regulator fields in the entropy of the quantum atmosphere the brick wall can be removed ²¹. The entropy then is proportional to certain power of the UV regulator, $S_q \sim A\mu^{d-2}$.

In our story of black hole relaxation the brick wall indeed gives the wanted effect: once the brick wall has been introduced the quasi-normal modes disappear completely and are replaced by a set of the real (normal) modes. This happens because the effective infinite size region near horizon is now removed and the whole space is the finite size region between

the brick wall and the boundary at spatial infinity. In such a system we expect periodicity with the period set by the brick wall parameter ϵ as $t_{\text{bw}} \sim 1/T \ln(1/\epsilon)$. This periodicity shows up in the boundary CFT correlation functions rather naturally. Indeed, these correlation functions are constructed from the bulk Green's function which describes propagation of the perturbation between two points on the boundary through the bulk. In the present case the perturbation from a point ϕ on the boundary goes along null-geodesic through the bulk, reflects at the brick wall and returns to the same point ϕ on the boundary. The time which the perturbation travels gives the periodicity for the boundary theory and it equals t_{bw} . Matching t_{bw} and $1/L_{\text{eff}}$ gives the relation between brick wall regulator ϵ and parameter k of the large N boundary CFT.

This probably should be enough for the explaining and reproducing the second time scale of the boundary CFT from the gravity side. The time t_{bw} is however much smaller than the Poincaré recurrence time which is expected to be of the order, $t_{\text{p}} \sim e^{\frac{A}{\epsilon}}$. So how to get this time scale in the model with the brick wall? We notice that the brick wall should not be ideally spherical. The possible complexity of the shape is not restricted. It may even be fractal. In order to serve as a regulator for the quantum entropy calculation brick wall should just stay at mean distance ϵ from the horizon but its shape can be arbitrary. For the recurrence time the shape is however crucial. In the absence of the spherical symmetry the perturbation emitted from the point ϕ on the boundary (which is still a circle) at spatial infinity goes along null-geodesic through the bulk, reflects from the brick wall, goes back and arrives at completely different point ϕ' on the boundary at spatial infinity. Only after a number of back and forth goings between two boundaries the perturbation can manage to arrive on the boundary at the same point where it was initially emitted. This number can be very large and it sets the periodicity for the boundary theory. The emerging geometric picture is standard set up for the system having classical chaos. Indeed, generic deviations from the spherical symmetry of one of the boundaries leads to chaotic behavior of the geodesics. This means that the 2-point functions on the boundary would generically have chaotic time evolution. The optical volume V between two boundaries seems to be the right quantity to measure the size of the phase space of the chaotic geodesics. Since $S_q \sim V$ the recurrence time $t_{\text{p}} \sim e^V$ gives the right estimate for the Poincaré time. In this picture the information sent to black hole eventually comes back. The characteristic time during which it should happen is set by the Poincaré recurrence time t_{p} .

The classical chaos of the geodesics manifests in the (normal) frequencies. The latter are the eigen values of the Laplace-type operator considered on the classical geometry. As we know from the relation between classical and quantum chaos the chaos of the geodesics in the classical system manifests in that the eigen values of the quantum problem are randomly distributed. Thus, the normal frequencies will be random numbers. This again means that the 2-point function on the boundary (we expect that the normal modes are still poles in the momentum representation of the correlation function) is chaotic function of time.

The irregularity of the shape of the brick wall may actually be physically meaningful. It can model the fluctuating quantum horizon. It may also be a way of representing the so-called stretched horizon (see ²²).

4.2. *Worm-hole modification: BTZ_k*

The horizon can be removed in a smooth way by modifying the black hole geometry and making it look like a worm-hole. As an example we present here a modification of the BTZ metric (1),

$$ds^2 = -(\sinh^2 y + \frac{1}{k^2}) dt^2 + dy^2 + \cosh^2 y d\phi^2, \quad (10)$$

which we call BTZ_k. The horizon which used to stay at $y = 0$ disappears in metric (10) if k is finite. The whole geometry now is that of worm-hole with the second asymptotic region at $y < 0$. The two asymptotic regions separated by horizon in classical BTZ metric can now talk to each other leaking the information through the narrow throat. The metric (10) is still asymptotically AdS although it is no more a constant curvature space-time. The Ricci scalar

$$R = -\frac{2}{(k^2 \sinh^2 y + 1)^2} [(k^2 + 1) + 3k^4 \sinh^4 y + 5k^2 \sinh^2 y] \quad (11)$$

approaches value -6 at infinite y and $-2(k^2 + 1)$ at $y = 0$ where the horizon used to stay. The normal frequencies in the space-time with metric (10) are real and are determined by the normalizability and the Dirichlet boundary condition at both spatial infinities. Since the space-time (10) is asymptotically AdS one can use the rules of the AdS/CFT duality and calculate the boundary correlation function. Technically it is more difficult than in the standard BTZ case since (10) is not maximally symmetric space. But the result should be a periodic in time function with the period set by parameter k . It would be interesting to do this calculation and see if this correlation function makes sense from the point of view of the expected

behavior of the boundary CFT at finite k . One can calculate the entropy of the thermal atmosphere in the metric (10). It is now finite with no need for introducing the brick wall. The entropy then behaves as $S_q \sim kA$ that is the right answer for the Bekenstein-Hawking entropy of BTZ black hole. Thus, the modification (10) gives us the right entropy and solves the unitarity problem.

Acknowledgments I would like to thank D. Birmingham and I. Sachs for enjoyable collaboration and many useful discussions. I also thank G. Arutyunov, J. Barbon, A. Morozov and N. Kaloper for important discussions.

References

1. A.L. Fetter and J.D. Walecka, "Quantum Theory of Many-Particle Systems," *McGraw-Hill Book Company* (1971).
2. S. W. Hawking, Phys. Rev. D **14** (1976) 2460.
3. L. Susskind, arXiv:hep-th/0204027.
4. O. Aharony, S.S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, Phys. Rep. **323**, 183 (2000).
5. G. T. Horowitz and V. E. Hubeny, Phys. Rev. D **62**, 024027 (2000).
6. D. Birmingham, I. Sachs and S. N. Solodukhin, Phys. Rev. D **67**, 104026 (2003).
7. J. L. F. Barbon and E. Rabinovici, JHEP **0311**, 047 (2003).
8. I. Sachs, Fortsch. Phys. **52**, 667 (2004).
9. J. L. F. Barbon and E. Rabinovici, Fortsch. Phys. **52**, 642 (2004).
10. V. Cardoso and J. P. S. Lemos, Phys. Rev. D **63**, 124015 (2001); D. Birmingham, Phys. Rev. D **64**, 064024 (2001).
11. D. Birmingham, I. Sachs and S.N. Solodukhin, Phys. Rev. Lett. **88**, 151301 (2002).
12. S. W. Hawking and D. N. Page, Commun. Math. Phys. **87**, 577 (1983).
13. P. Di Francesco, P. Mathieu and D. Senechal, "Conformal Field Theory," *New York, USA: Springer* (1997).
14. S. Carlip and C. Teitelboim, Phys. Rev. D **51**, 622 (1995).
15. J. M. Maldacena and A. Strominger, JHEP **9812**, 005 (1998).
16. J. M. Maldacena, arXiv:hep-th/0106112.
17. L. Chekhov, arXiv:hep-th/9811146; E. Keski-Vakkuri, Phys. Rev. D **59**, 104001 (1999).
18. V. Balasubramanian, P. Kraus and A. E. Lawrence, Phys. Rev. D **59**, 046003 (1999).
19. U. H. Danielsson, E. Keski-Vakkuri and M. Kruczenski, Nucl. Phys. B **563**, 279 (1999).
20. G. 't Hooft, Nucl. Phys. B **256**, 727 (1985).
21. J. G. Demers, R. Lafrance and R. C. Myers, Phys. Rev. D **52**, 2245 (1995).
22. N. Iizuka, D. Kabat, G. Lifschytz and D. A. Lowe, Phys. Rev. D **68**, 084021 (2003).